

diffops_tutorial

Exercise 1.

```
y = function('y')(x)
```

```
de=diff(y,x) + y
```

```
desolve(de,y)
```

```
_C*e^(-x)
```

```
desolve(de==e^x,y)
```

```
1/2*(2*_C + e^(2*x))*e^(-x)
```

Exercise 3.

Define the ring of differential operators with rational coefficients

```
from ore_algebra import *
```

```
R.<x> = PolynomialRing(QQ)
```

```
K=R.fraction_field()
```

```
B.<Dx>=OreAlgebra(K)
```

```
type(B)
```

```
<class 'ore_algebra.ore_algebra.OreAlgebra_generic'>
```

Multiplication of differential operators is noncommutative

```
Dx*x
```

```
x*Dx + 1
```

Different factorizations from the lecture

```
(Dx+1/(x+2))*(Dx-1/(x+2))
```

```
Dx^2
```

The first example

```
Dx^2+(3/x)*Dx+(1/x^2)
```

```
Dx^2 + 3/x*Dx + 1/x^2
```

and a factorization

```
(Dx + 2/x)*(Dx+1/x)
```

$$Dx^2 + 3/x*Dx + 1/x^2$$

Second order differential operator

$$L=Dx^2 + ((-x + 1)/x)*Dx + (-x - 1)/x^2$$

L(y)

$$-(x - 1)*diff(y(x), x)/x - (x + 1)*y(x)/x^2 + diff(y(x), x, x)$$

desolve(L(y),y)