

Tutorial session 3: Solving in Computer Algebra
Dr. Daniel Duviol Tcheutia
October 9, 2017, Douala, Cameroon

1. Solve the system of equations

$$\begin{cases} x^2 + y^2 = 5 \\ xy = y^2 - 2, \end{cases}$$

the equation $|x| = 7$, and the inequalities $x^2 - 2x - 3 < 0$, $x^2 - 3x + 1 < 0$ and $|x - 3| \cdot |3 - x| > |x|$.
Hint: One can use the command `solve`, `to_poly_solve` after loading `load(to_poly_solve)`, and `solve_rat_ineq` after loading `load(solve_rat_ineq)`

2. Solve the differential equations

$$\begin{aligned} xy' &= y \ln(xy) - y, \quad y(1) = e, \\ y'' + 5y' + 6y &= 0, \quad y(0) = 2, \quad y'(0) = 3; \end{aligned}$$

the Legendre polynomial differential equation

$$(1 - x^2)y'' - 2xy' + n(n + 1)y = 0;$$

and the system of differential equations

$$\begin{cases} y_1'(x) = 2y_1(x) - 2y_2(x) \\ y_2'(x) = -2y_1(x) + 2y_3(x) \\ y_3'(x) = 2y_2(x) + 2y_3(x) \\ y_1(0) = 2, \quad y_2(0) = 1, \quad y_3(0) = -2. \end{cases}$$

Hint: You can use the commands `desolve`, `ode2`, `ic1`, `ic2`, `atvalue`, `contrib_ode` after loading `load(contrib_ode)`.

3. Solve the recurrence equation

$$F(n) = -3F(n - 1) - 2F(n - 2), \quad F(1) = 1, \quad F(2) = 1.$$

4. Solve the equation $7x \equiv 1 \pmod{218}$ and the system of equations

$$\begin{cases} x \equiv 2 \pmod{12} \\ x \equiv 8 \pmod{10} \\ x \equiv 10 \pmod{14}. \end{cases}$$

Hint: You can use `inv_mod` or `power_mod` and `chinese`.