Tutorial session 1: Introduction to Maxima
Dr. Daniel Duviol Tcheutia
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1. Compute $h = \frac{(x^2+2^3\pi)(1+\sqrt{x})}{7}$ for $x = 3$ and $x = 9$. Give the numerical approximation of the result with 10 and 25 digit precision, respectively.

2. Write $\cos^9(t) - \sin^9(t)$ as a Fourier polynomial

$$a_0 + \sum_{k=1}^{N} a_k \cos(kt) + \sum_{k=1}^{N} b_k \sin(kt),\ N \in \mathbb{N}.$$

   In converse, write the output as a polynomial of the variables $\cos(t)$ and $\sin(t)$. Hint: you can use the commands `trigreduce, trigexpand, trigsimp`

3. Consider the rational expression

$$r = \frac{x^4 + x^3 - 4x^2 - 4x}{x^4 + x^3 - x^2 - x}.$$

   Determine its normal form, its factorized form and its partial fraction decomposition. Hint: you can use the commands `ratsimp, factor, partfrac`

4. Define the function

$$f : (x, y, z) \longmapsto \frac{z}{x^2 + y^2 + z^2},$$

   and check that f is solution of the differential equation

$$\frac{\partial^2}{\partial x \partial y} f + \frac{4x}{x^2 + y^2 + z^2} \frac{\partial}{\partial y} f = 0.$$

5. Compute the following integrals:

$$\int \sqrt{x^2 - a^2} dx;\ \int_0^\infty \frac{\sin x}{x} dx;\ \int_0^\infty x e^{-ax} \cos(wx) dx \text{ for } a > 0.$$

6. Plot on [1/3, 1] the graph of

$$f(x) = e^x - \frac{1}{x},$$

   and use the `find_root` command to find a numerical approximation of the solution to the equation $f(x) = 0$.

7. Determine the following sums:

$$\sum_{k=0}^{\infty} \binom{n}{k},\ \sum_{k=0}^{\infty} a q^k \text{ for } |q| < 1 \text{ and for } |q| > 1,\ \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{k!};\ \sum_{k=0}^{\infty} \binom{n}{k} x^k y^{-n-k}.$$

   Hint: Use `simpsum` or `simplify_sum` after loading `load(simplify_sum)` to simplify the sum, `assume` for the assumption and `forget` to forget the assumption.

8. Compute the Taylor polynomial at $x_0 = 0$ of order 8 of the function

$$f(x) = \frac{\ln(x+1) - \tan(x) + \frac{1}{2}\sin^2(x)}{3x^2 \sin^2(x)},$$

   and deduce the limit of $f(x)$ as $x \to 0$.

9. Compute the following limits:

$$\lim_{x\to 0}(\cos x)^{\frac{1}{x^3}};\ \lim_{x\to\infty}(2^x+3^x)^{\frac{1}{x}};\ \lim_{n\to\infty}\frac{n!}{n^n e^{-n}\sqrt{2\pi n}},\ \lim_{x\to\infty} e^{-ax}\cos(bx) \text{ assuming } a>0.$$

10. Consider the matrix

$$M=\begin{pmatrix} 0 & -\frac{4}{5} & -\frac{3}{5} \\ \frac{4}{5} & -\frac{9}{25} & \frac{12}{25} \\ \frac{3}{5} & \frac{12}{25} & -\frac{16}{25} \end{pmatrix}$$

defined in `Maxima` by `M: matrix([0,-4/5,-3/5],[4/5,-9/25,12/25],[3/5,12/25,-16/25])`. Determine the trace, determinant, the rank, the inverse, the transpose, the characteristic polynomial, the eigenvalues, the eigenvectors, the matrix exponential $e^M=\sum_{n=0}^{\infty}\frac{A^n}{n!}$ of the matrix M. Hint: you can use the commands `matrix`, `length`, `determinant`, `rank`, `invert`, `transpose`, `charpoly`, `eigenvalues`, `eigenvectors`, `matrixexp`.